

Exercise 2.1 (Revised) - Chapter 2 - Polynomials - Ncert Solutions class 9 - Maths

Updated On 11-02-2025 By Lithanya

NCERT Solutions Class 9 Maths Chapter 2 - Polynomials | Comprehensive Guide

Ex 2.1 Question 1.

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Answer.

(i) $4x^2 - 3x + 7$

We can observe that in the polynomial $4x^2 - 3x + 7$, we have x as the only variable and the powers of x in each term are a whole number.

Therefore, we conclude that $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

We can observe that in the polynomial $y^2 + \sqrt{2}$, we have y as the only variable and the powers of y in each term are a whole number.

Therefore, we conclude that $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

We can observe that in the polynomial $3\sqrt{t} + t\sqrt{2}$, we have t as the only variable and the powers of t in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t} + t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y + \frac{2}{y}$

We can observe that in the polynomial $y + \frac{2}{y}$, we have y as the only variable and the powers of y in each term are not a whole number.

Therefore, we conclude that $y + \frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10} + y^3 + t^{30}$

We can observe that in the polynomial $x^{10} + y^3 + t^{30}$, we have x, y and t as the variables and the powers of x, y and t in each term is a whole number.

Therefore, we conclude that $x^{10} + y^3 + t^{30}$ is a polynomial but not a polynomial in one variable.

Ex 2.1 Question 2.

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Answer.

(i) $2 + x^2 + x$

The coefficient of x^2 in the polynomial $2 + x^2 + x$ is 1 .

(ii) $2 - x^2 + x^3$

The coefficient of x^2 in the polynomial $2 - x^2 + x^3$ is -1 .

(iii) $\frac{\pi}{2}x^2 + x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x - 1$

The coefficient of x^2 in the polynomial $\sqrt{2}x - 1$ is 0 .

Ex 2.1 Question 3.

Give one example each of a binomial of degree 35 , and of a monomial of degree 100 .

Answer.

The binomial of degree 35 can be $x^{35} + 9$.

The binomial of degree 100 can be t^{100} .

Ex 2.1 Question 4.

Write the degree of each of the following polynomials:

(i) $p(x) = 5x^3 + 4x^2 + 7x$

(ii) $p(y) = 4 - y^2$

(iii) $f(t) = 5t - \sqrt{7}$

(iv) $f(x) = 3$

Answer.

(i) $5x^3 + 4x^2 + 7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $5x^3 + 4x^2 + 7x$, the highest power of the variable x is 3 .

Therefore, we conclude that the degree of the polynomial $5x^3 + 4x^2 + 7x$ is 3 .

(ii) $4 - y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $4 - y^2$, the highest power of the variable y is 2 .

Therefore, we conclude that the degree of the polynomial $4 - y^2$ is 2 .

(iii) $5t - \sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We observe that in the polynomial $5t - \sqrt{7}$, the highest power of the variable t is 1 .

Therefore, we conclude that the degree of the polynomial $5t - \sqrt{7}$ is 1 .

(iv) 3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3 , the highest power of the assumed variable x is 0 .

Therefore, we conclude that the degree of the polynomial 3 is 0 .

Ex 2.1 Question 5.

Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

Answer.

(i) $x^2 + x$

We can observe that the degree of the polynomial $x^2 + x$ is 2 .

Therefore, we can conclude that the polynomial $x^2 + x$ is a quadratic polynomial.

(ii) $x - x^3$

We can observe that the degree of the polynomial $x - x^3$ is 3 .



Therefore, we can conclude that the polynomial $x - x^3$ is a cubic polynomial.

(iii) $y + y^2 + 4$

We can observe that the degree of the polynomial $y + y^2 + 4$ is 2 .

Therefore, the polynomial $y + y^2 + 4$ is a quadratic polynomial.

(iv) $1 + x$

We can observe that the degree of the polynomial $(1 + x)$ is 1 .

Therefore, we can conclude that the polynomial $1 + x$ is a linear polynomial.

(v) $3t$

We can observe that the degree of the polynomial $(3t)$ is 1 .

Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.

(vi) r^2

We can observe that the degree of the polynomial r^2 is 2 .

Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.

(vii) $7x^3$

We can observe that the degree of the polynomial $7x^3$ is 3 .

Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

Exercise 2.2 (Revised) - Chapter 2 - Polynomials - Ncert Solutions class 9 - Maths

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Ex 2.2 Question 1.

Find the value of the polynomial $5x - 4x^2 + 3$ at

- (i) $x = 0$
- (ii) $x = -1$
- (iii) $x = 2$

Answer.

(i) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get $f(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

Therefore, we conclude that at $x = 0$, the value of the polynomial $5x - 4x^2 + 3$ is 3 .

(ii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute -1 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get. $f(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -6$

Therefore, we conclude that at $x = -1$, the value of the polynomial $5x - 4x^2 + 3$ is -6

(iii) Let $f(x) = 5x - 4x^2 + 3$.

We need to substitute 0 in the polynomial $f(x) = 5x - 4x^2 + 3$ to get $f(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = -3$

Therefore, we conclude that at $x = 2$, the value of the polynomial $5x - 4x^2 + 3$ is -3 .

Ex 2.2 Question 2.

Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

- (i) $p(y) = y^2 - y + 1$
- (ii) $p(t) = 2 + t + 2t^2 - t^3$
- (iii) $p(x) = x^3$
- (iv) $p(x) = (x - 1)(x + 1)$

Answer.

(i) $p(y) = y^2 - y + 1$

At $p(0)$:

$$p(0) = (0)^2 - 0 + 1 = 1$$

At $p(1)$:

$$p(1) = (1)^2 - 1 + 1 = 1 - 0 = 1$$



At $p(2)$:

$$p(2) = (2)^2 - 2 + 1 = 4 - 1 = 3$$

$$(ii) \ p(t) = 2 + t + 2t^2 - t^3$$

At $p(0)$

$$p(0) = 2 + (0) + 2(0)^2 - (0)^3 = 2$$

At $p(1)$

$$p(1) = 2 + (1) + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4$$

At $p(2)$

$$p(2) = 2 + (2) + 2(2)^2 - (2)^3 = 4 + 8 - 8 = 4$$

$$(iii) \ p(x) = (x)^3$$

$$\text{At } p(0) \ p(0) = (0)^3 = 0$$

At $p(1)$

$$p(1) = (1)^3 = 1$$

At $p(2)$

$$p(2) = (2)^3 = 8$$

$$(vi) \ p(x) = (x - 1)(x + 1)$$

At $p(0)$:

$$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$$

At $p(1)$

$$p(1) = (1 - 1)(2 + 1) = (0)(3) = 0$$

At $p(2)$:

$$p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$$

Ex 2.2 Question 3.

Verify whether the following are zeroes of the polynomial, indicated against them.

$$(i) \ p(x) = 3x + 1, \quad x = -\frac{1}{3}$$

$$(ii) \ p(x) = 5x - \pi, \quad x = \frac{4}{5}$$

$$(iii) \ p(x) = x^2 - 1, \quad x = -1, 1$$

$$(iv) \ p(x) = (x + 1)(x - 2), \quad x = -1, 2$$

$$(v) \ p(x) = x^2, \quad x = 0$$

$$(vi) \ p(x) = lx + m, \quad x = -\frac{m}{l}$$

$$(vii) \ p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

$$(viii) \ p(x) = 2x + 1, \quad x = -\frac{1}{2}$$

Answer.

$$(i) \ p(x) = 3x + 1, \quad x = -\frac{1}{3}$$

We need to check whether $p(x) = 3x + 1$ at $x = -\frac{1}{3}$ is equal to zero or not.

$$p\left(-\frac{1}{3}\right) = 3x + 1 = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$$

Therefore, we can conclude that $x = -\frac{1}{3}$ is a zero of the polynomial $p(x) = 3x + 1$.

$$(ii) \ p(x) = 5x - \pi, \quad x = \frac{4}{5}$$

We need to check whether $p(x) = 5x - \pi$ at $x = \frac{4}{5}$ is equal to zero or not.

$$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$$

Therefore, $x = \frac{4}{5}$ is not a zero of the polynomial $p(x) = 5x - \pi$.

$$(iii) \ p(x) = x^2 - 1, \quad x = -1, 1$$

We need to check whether $p(x) = x^2 - 1$ at $x = -1, 1$ is equal to zero or not.

At $x = -1$

$$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$$

At $x = 1$

$$p(1) = (1)^2 - 1 = 1 - 1 = 0$$

Therefore, $x = -1, 1$ are the zeros of the polynomial $p(x) = x^2 - 1$.

$$(iv) \ p(x) = (x + 1)(x - 2), \quad x = -1, 2$$

We need to check whether $p(x) = (x + 1)(x - 2)$ at $x = -1, 2$ is equal to zero or not.

At $x = -1$

$$p(-1) = (-1 + 1)(-1 - 2) = (0)(-3) = 0$$

At $x = 2$

$$p(2) = (2 + 1)(2 - 2) = (3)(0) = 0$$

Therefore, $x = -1, 2$ are the zeros of the polynomial $p(x) = (x + 1)(x - 2)$.

(v) $p(x) = x^2, x = 0$

We need to check whether $p(x) = x^2$ at $x = 0$ is equal to zero or not.

$$p(0) = (0)^2 = 0$$

Therefore, we can conclude that $x = 0$ is a zero of the polynomial $p(x) = x^2$.

(vi) $p(x) = lx + m, x = -\frac{m}{l}$

We need to check whether $p(x) = +m = 0$

$$p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = m + m = 0$$

Therefore $x = -\frac{m}{l}$ is a zero of the polynomial $p(x) = lx + m$.

(vii)

$$p(x) = 3x^2 - 1, \quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$$

We need to check whether

$p(x) = 3x^2 - 1$ at $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ is equal to zero or not.

$$x = \frac{-1}{\sqrt{3}}$$

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$$

$$x = \frac{2}{\sqrt{3}}$$

At

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$$

Therefore, we can conclude that $x = \frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x) = 3x^2 - 1$ but $x = \frac{2}{\sqrt{3}}$ is not a zero of the polynomial

$$p(x) = 3x^2 - 1.$$

(viii)

$$p(x) = 2x + 1, x = -\frac{1}{2}$$

We need to check whether $p(x) = 2x + 1$ at $x = -\frac{1}{2}$ is equal to zero or not.

$$p\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 1 = -1 + 1 = 0$$

Therefore, $x = -\frac{1}{2}$ is a zero of the polynomial $p(x) = 2x + 1$

Ex 2.2 Question 4.

Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x) = 3x - 2$

(v) $p(x) = 3x$

(vi) $p(x) = ax, a \neq 0$

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Answer.

(i) $p(x) = x + 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = x + 5$ equal to 0, we get

$$x + 5 = 0 \Rightarrow x = -5$$

Therefore, we conclude that the zero of the polynomial $p(x) = x + 5$ is -5.

(ii) $p(x) = x - 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = x - 5$ equal to 0, we get

$$x - 5 = 0 \Rightarrow x = 5$$

Therefore, we conclude that the zero of the polynomial $p(x) = x - 5$ is 5.

(iii) $p(x) = 2x + 5$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 2x + 5$ equal to 0, we get

$$2x + 5 = 0 \Rightarrow x = \frac{-5}{2}$$



Therefore, we conclude that the zero of the polynomial $p(x) = 2x + 5$ is $-\frac{5}{2}$.

(iv) $p(x) = 3x - 2$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 3x - 2$ equal to 0, we get

$$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$$

Therefore, we conclude that the zero of the polynomial $p(x) = 3x - 2$ is $\frac{2}{3}$.

(v) $p(x) = 3x$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = 3x$ equal to 0, we get

$$3x = 0 \Rightarrow x = 0$$

Therefore, we conclude that the zero of the polynomial $p(x) = 3x$ is 0.

(vi) $p(x) = ax, a \neq 0$

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = ax$ equal to 0, we get

$$ax = 0 \Rightarrow x = 0$$

Therefore, we conclude that the zero of the polynomial $p(x) = ax, a \neq 0$ is 0.

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

$ax + b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = cx + d$ equal to 0, we get

$$cx + d = 0$$

$$\Rightarrow x = -\frac{d}{c}$$

Therefore, we conclude that the zero of the polynomial $p(x) = cx + d, c \neq 0, c, d$ are real numbers. is $-\frac{d}{c}$.



Exercise 2.3 (Revised) - Chapter 2 - Polynomials - Ncert Solutions class 9 - Maths

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Exercise 2.3 Question 1.

Determine which of the following polynomials has $(x+1)$ a factor:

- (i) $x^3 + x^2 + x + 1$
- (ii) $x^4 + x^3 + x^2 + x + 1$
- (iii) $x^4 + 3x^3 + 3x^2 + x + 1$
- (iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Answer.

(i) $x^3 + x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^3 + x^2 + x + 1$$

$$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

$$= -1 + 1 - 1 + 1$$

$$= 0$$

We conclude that on dividing the polynomial $x^3 + x^2 + x + 1$ by $(x + 1)$, we get the remainder as 0.

Therefore, we conclude that $(x + 1)$ is a factor of $x^3 + x^2 + x + 1$.

(ii) $x^4 + x^3 + x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^4 + x^3 + x^2 + x + 1$$

$$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 - 1 + 1 - 1 + 1$$

$$= 1$$

We conclude that on dividing the polynomial $x^4 + x^3 + x^2 + x + 1$ by $(x + 1)$, we will get the remainder as 1, which is not 0 .

Therefore, we conclude that $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

While applying the factor theorem, we get

$$p(x) = x^4 + 3x^3 + 3x^2 + x + 1$$

$$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 - 3 + 3 - 1 + 1$$

$$= 1$$

We conclude that on dividing the polynomial $x^4 + 3x^3 + 3x^2 + x + 1$ by $(x + 1)$, we will get the remainder as 1 , which is not 0 .

Therefore, we conclude that $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$



While applying the factor theorem, we get

$$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$$

$$\begin{aligned} p(-1) &= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} \\ &= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

We conclude that on dividing the polynomial $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$ by $(x + 1)$, we will get the remainder as $2\sqrt{2}$, which is not 0 .

Therefore, we conclude that $(x + 1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

Exercise 2.3 Question 2.

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Answer.

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-1) = 0$.

$$\begin{aligned} p(-1) &= 2(-1)^3 + (-1)^2 - 2(-1) - 1 \\ &= 2 + 1 - 1 - 2 \\ &= 0 \end{aligned}$$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-2) = 0$.

$$\begin{aligned} p(-2) &= (-2)^3 + 3(-2)^2 + 3(-2) + 1 \\ &= -8 + 12 - 6 + 1 \\ &= -1 \end{aligned}$$

Therefore, we conclude that the $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

We know that according to the factor theorem, $(x - a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(3) = 0$.

$$\begin{aligned} p(3) &= (3)^3 - 4(3)^2 + (3) + 6 \\ &= 27 - 36 + 3 + 6 \\ &= 0 \end{aligned}$$

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

Exercise 2.3 Question 3.

Find the value of k , if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Answer.

(i) $p(x) = x^2 + x + k$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x)$$

We conclude that if $(x - 1)$ is a factor of $p(x) = x^2 + x + k$, then $p(1) = 0$.

$$\begin{aligned} p(1) &= (1)^2 + (1) + k = 0, \text{ or} \\ k + 2 &= 0 \end{aligned}$$

$$k = -2$$

Therefore, we can conclude that the value of k is -2 .

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x - 1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then $p(1) = 0$.

$$p(1) = 2(1)^2 + k(1) + \sqrt{2} = 0, \text{ or}$$

$$2 + k + \sqrt{2} = 0$$

$$k = -(2 + \sqrt{2}).$$

Therefore, we can conclude that the value of k is $-(2 + \sqrt{2})$.

$$(iii) p(x) = kx^2 - \sqrt{2}x + 1$$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x).$$

We conclude that if $(x - 1)$ is a factor of $p(x) = kx^2 - \sqrt{2}x + 1$, then $p(1) = 0$.

$$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0 \text{ or}$$

$$k - \sqrt{2} + 1 = 0$$

$$k = \sqrt{2} - 1$$

Therefore, we can conclude that the value of k is $\sqrt{2} - 1$.

$$(iv) p(x) = kx^2 - 3x + k$$

We know that according to the factor theorem

$$p(a) = 0, \text{ if } x - a \text{ is a factor of } p(x)$$

We conclude that if $(x - 1)$ is a factor of $p(x) = kx^2 - 3x + k$, then $p(1) = 0$.

$$p(1) = k(1)^2 - 3(1) + k; \text{ or } 2k - 3 = 0 \Rightarrow k = \frac{3}{2}$$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

Exercise 2.3 Question 4.

Factorize:

$$(i) 12x^2 - 7x + 1$$

$$(ii) 2x^2 + 7x + 3$$

$$(iii) 6x^2 + 5x - 6$$

$$(iv) 3x^2 - x - 4$$

Answer.

$$(i) 12x^2 - 7x + 1$$

$$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$$

$$= 3x(4x - 1) - 1(4x - 1)$$

$$= (3x - 1)(4x - 1).$$

Therefore, we conclude that on factorizing the polynomial $12x^2 - 7x + 1$, we get $(3x - 1)(4x - 1)$.

$$(ii) 2x^2 + 7x + 3$$

$$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$$

$$= 2x(x + 3) + 1(x + 3)$$

$$= (2x + 1)(x + 3).$$

Therefore, we conclude that on factorizing the polynomial $2x^2 + 7x + 3$, we get $(2x + 1)(x + 3)$.

$$(iii) 6x^2 + 5x - 6$$

$$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$

$$= 3x(2x + 3) - 2(2x + 3)$$

$$= (3x - 2)(2x + 3).$$

Therefore, we conclude that on factorizing the polynomial $6x^2 + 5x - 6$, we get $(3x - 2)(2x + 3)$.

$$(iv) 3x^2 - x - 4$$

$$3x^2 - x - 4 = 3x^2 + 3x - 4x - 4$$

$$= 3x(x + 1) - 4(x + 1)$$

$$= (3x - 4)(x + 1)$$

Therefore, we conclude that on factorizing the polynomial $3x^2 - x - 4$, we get $(3x - 4)(x + 1)$.

Exercise 2.3 Question 5.

Factorize:

$$(i) x^3 - 2x^2 - x + 2$$

$$(ii) x^3 - 3x^2 - 9x - 5$$

$$(iii) x^3 + 13x^2 + 32x + 20$$

$$(iv) 2y^3 + y^2 - 2y - 1$$

Answer.

$$(i) x^3 - 2x^2 - x + 2$$

We need to consider the factors of 2, which are $\pm 1, \pm 2$.

Let us substitute 1 in the polynomial $x^3 - 2x^2 - x + 2$, to get
 $(1)^3 - 2(1)^2 - (1) + 2 = 1 - 1 - 2 + 2 = 0$

Thus, according to factor theorem, we can conclude that $(x - 1)$ is a factor of the polynomial $x^3 - 2x^2 - x + 2$.

Let us divide the polynomial $x^3 - 2x^2 - x + 2$ by $(x - 1)$, to get

$$\begin{array}{r} \overline{) \begin{array}{r} x^3-2x^2-x+2 \\ x^3-x^2 \\ \hline -x^2-x \\ -x^2+x \\ \hline -2x+2 \\ -2x+2 \\ \hline 0 \end{array}} \\ \overline{) x^3-2x^2-x+2} \\ \underline{x^3-x^2} \\ -x \\ \underline{-x^2+x} \\ 2 \\ \underline{-2x+2} \\ 0 \end{array}$$

$$\begin{aligned} x^3 - 2x^2 - x + 2 &= (x - 1) (x^2 - x - 2) . \\ x^3 - 2x^2 - x + 2 &= (x - 1) (x^2 - x - 2) . \\ &= (x - 1) (x^2 + x - 2x - 2) \\ &= (x - 1)[x(x + 1) - 2(x + 1)] \\ &= (x - 1)(x - 2)(x + 1). \end{aligned}$$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$, we get $(x - 1)(x - 2)(x + 1)$.
(ii) $x^3 - 3x^2 - 9x - 5$

We need to consider the factors of -5 , which are $\pm 1, \pm 5$.

Let us substitute 1 in the polynomial $x^3 - 3x^2 - 9x - 5$, to get
 $(-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = 0$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3 - 3x^2 - 9x - 5$.

Let us divide the polynomial $x^3 - 3x^2 - 9x - 5$ by $(x + 1)$, to get

$$\begin{array}{r} \overline{) \begin{array}{r} x^3-3x^2-9x-5 \\ x^3+x^2 \\ \hline -4x^2-9x \\ -4x^2-4x \\ \hline -5x-5 \\ -5x-5 \\ \hline 0 \end{array}} \\ \overline{) x^3-3x^2-9x-5} \\ \underline{x^3+x^2} \\ -4x^2-9x \\ \underline{-4x^2-4x} \\ -5x \\ \underline{-5x-5} \\ 0 \end{array}$$

$$\begin{aligned} x^3 - 3x^2 - 9x - 5 &= (x + 1) (x^2 - 4x - 5) \\ &= (x + 1) (x^2 + x - 5x - 5) \\ &= (x + 1)[x(x + 1) - 5(x + 1)] \\ &= (x + 1)(x - 5)(x + 1). \end{aligned}$$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 3x^2 - 9x - 5$, we get $(x + 1)(x - 5)(x + 1)$
(iii) $x^3 + 13x^2 + 32x + 20$

We need to consider the factors of 20 , which are $\pm 5, \pm 4, \pm 2, \pm 1$.

Let us substitute -1 in the polynomial $x^3 + 13x^2 + 32x + 20$, to get
 $(-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20 = -20 + 20 = 0$

$$\begin{array}{r} \overline{) \begin{array}{r} x^3+13x^2+32x+20 \\ x^3+x^2 \\ \hline 12x^2+32x \\ 12x^2+12x \\ \hline 20x+20 \\ 20x+20 \\ \hline 0 \end{array}} \\ \overline{) x^3+13x^2+32x+20} \\ \underline{x^3+x^2} \\ 12x^2+32x \\ \underline{12x^2+12x} \\ 20x+20 \\ \underline{20x+20} \\ 0 \end{array}$$

$$\begin{aligned}
 x^3 + 13x^2 + 32x + 20 &= (x + 1)(x^2 + 12x + 20) \\
 &= (x + 1)(x^2 + 2x + 10x + 20) \\
 &= (x + 1)[x(x + 2) + 10(x + 2)] \\
 &= (x + 1)(x + 10)(x + 2).
 \end{aligned}$$

Therefore, we can conclude that on factorizing the polynomial $x^3 + 13x^2 + 32x + 20$, we get $(x + 1)(x - 10)(x + 2)$

(iv) $2y^3 + y^2 - 2y - 1$

We need to consider the factors of -1, which are ± 1 .

Let us substitute 1 in the polynomial $2y^3 + y^2 - 2y - 1$, to get

$$2(1)^3 + (1)^2 - 2(1) - 1 = 2 + 1 - 2 - 1 = 3 - 3 = 0$$

Thus, according to factor theorem, we can conclude that $(y - 1)$ is a factor of the polynomial $2y^3 + y^2 - 2y - 1$.

Let us divide the polynomial $2y^3 + y^2 - 2y - 1$ by $(y - 1)$, to get

$$\begin{array}{r}
 2y^2 + 3y + 1 \\
 y - 1 \overline{) 2y^3 + y^2 - 2y - 1} \\
 \underline{2y^3 - 2y^2} \\
 3y^2 - 2y \\
 \underline{3y^2 - 3y} \\
 y - 1 \\
 \underline{y - 1} \\
 0
 \end{array}$$

$$\begin{aligned}
 2y^3 + y^2 - 2y - 1 &= (y - 1)(2y^2 + 3y + 1) \\
 &= (y - 1)(2y^2 + 2y + y + 1) \\
 &= (y - 1)[2y(y + 1) + 1(y + 1)] \\
 &= (y - 1)(2y + 1)(y + 1).
 \end{aligned}$$

Therefore, we can conclude that on factorizing the polynomial $2y^3 + y^2 - 2y - 1$, we get $(y - 1)(2y + 1)(y + 1)$.



Exercise 2.4 (Revised) – Chapter 2 – Polynomials – Ncert Solutions class 9 – Maths

Updated On 11-02-2025 By Lithanya

Chapter 2 – Polynomials | NCERT Solutions for Class 9 Maths

Ex 2.4 Question 1.

Use suitable identities to find the following products:

- (i) $(x + 4)(x + 10)$
- (ii) $(x + 8)(x - 10)$
- (iii) $(3x + 4)(3x - 5)$
- (iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$
- (v) $(3 - 2x)(3 + 2x)$

Answer.

(i) $(x + 4)(x + 10)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$.

We need to apply the above identity to find the product $(x + 4)(x + 10)$

$$\begin{aligned}(x + 4)(x + 10) &= x^2 + (4 + 10)x + (4 \times 10) \\ &= x^2 + 14x + 40\end{aligned}$$

Therefore, we conclude that the product $(x + 4)(x + 10)$ is $x^2 + 14x + 40$.

(ii) $(x + 8)(x - 10)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$.

We need to apply the above identity to find the product $(x + 8)(x - 10)$

$$\begin{aligned}(x + 8)(x - 10) &= x^2 + [8 + (-10)]x + [8 \times (-10)] \\ &= x^2 - 2x - 80.\end{aligned}$$

Therefore, we conclude that the product $(x + 8)(x - 10)$ is $x^2 - 2x - 80$.

(iii) $(3x + 4)(3x - 5)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$.

We need to apply the above identity to find the product $(3x + 4)(3x - 5)$

$$\begin{aligned}(3x + 4)(3x - 5) &= (3x)^2 + [4 + (-5)]3x + [4 \times (-5)] \\ &= 9x^2 - 3x - 20.\end{aligned}$$

Therefore, we conclude that the product $(3x + 4)(3x - 5)$ is $9x^2 - 3x - 20$.

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$



We know that $(x + y)(x - y) = x^2 - y^2$.

We need to apply the above identity to find the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

$$\begin{aligned} & \left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right) \\ &= (y^2)^2 - \left(\frac{3}{2}\right)^2 = y^4 - \frac{9}{4}. \end{aligned}$$

Therefore, we conclude that the product $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$ is $\left(y^4 - \frac{9}{4}\right)$.

(v) $(3 + 2x)(3 - 2x)$

We know that $(x + y)(x - y) = x^2 - y^2$.

We need to apply the above identity to find the product $(3 + 2x)(3 - 2x)$

$$\begin{aligned} (3 + 2x)(3 - 2x) &= (3)^2 - (2x)^2 \\ &= 9 - 4x^2. \end{aligned}$$

Therefore, we conclude that the product $(3 + 2x)(3 - 2x)$ is $(9 - 4x^2)$.

Ex 2.4 Question 2.

Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98×96

(iii) 104×96

Answer.

(i) 103×107

103×107 can also be written as $(100 + 3)(100 + 7)$.

We can observe that, we can apply the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\begin{aligned} (100 + 3)(100 + 7) &= (100)^2 + (3 + 7)(100) + 3 \times 7 \\ &= 10000 + 1000 + 21 \\ &= 11021 \end{aligned}$$

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100 - 5)(100 - 4)$

We can observe that, we can apply the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$\begin{aligned} (100 - 5)(100 - 4) &= (100)^2 + [(-5) + (-4)](100) + (-5) \times (-4) \\ &= 10000 - 900 + 20 \\ &= 9120 \end{aligned}$$

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100 + 4)(100 - 4)$.

We can observe that, we can apply the identity $(x + y)(x - y) = x^2 - y^2$ with respect to the expression $(100 + 4)(100 - 4)$, to get

$$\begin{aligned} (100 + 4)(100 - 4) &= (100)^2 - (4)^2 \\ &= 10000 - 16 \\ &= 9984 \end{aligned}$$

Therefore, we conclude that the value of the product 104×96 is 9984.

Ex 2.4 Question 3.

Factorize the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Answer.

(i) $9x^2 + 6xy + y^2$

$$9x^2 + 6xy + y^2 = (3x)^2 + 2 \times 3x \times y + (y)^2$$

We can observe that, we can apply the identity $(x + y)^2 = x^2 + 2xy + y^2$

$$\Rightarrow (3x)^2 + 2 \times 3x \times y + (y)^2 = (3x + y)^2.$$

(ii) $4y^2 - 4y + 1$

$$4y^2 - 4y + 1 = (2y)^2 - 2 \times 2y \times 1 + (1)^2$$

We can observe that, we can apply the identity $(x - y)^2 = x^2 - 2xy + y^2$

$$\Rightarrow (2y)^2 - 2 \times 2y \times 1 + (1)^2 = (2y - 1)^2.$$

(iii) $x^2 - \frac{y^2}{100}$



We can observe that, we can apply the identity $(x)^2 - (y)^2 = (x + y)(x - y)$

$$\Rightarrow (x)^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$$

Ex 2.4 Question 4.

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

Answer.

(i) $(x + 2y + 4z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(x + 2y + 4z)^2$.

$$\begin{aligned}(x + 2y + 4z)^2 &= (x)^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x \\ &= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx\end{aligned}$$

(ii) $(2x - y + z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(2x - y + z)^2$.

$$\begin{aligned}(2x - y + z)^2 &= [2x + (-y) + z]^2 \\ &= (2x)^2 + (-y)^2 + (z)^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x \\ &= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx\end{aligned}$$

(iii) $(-2x + 3y + 2z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 3y + 2z)^2$.

$$\begin{aligned}(-2x + 3y + 2z)^2 &= [(-2x) + 3y + 2z]^2 \\ &= (-2x)^2 + (3y)^2 + (2z)^2 + 2 \times (-2x) \times 3y + 2 \times 3y \times 2z + 2 \times 2z \times (-2x) \\ &= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx\end{aligned}$$

(iv) $(3a - 7b - c)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(3a - 7b - c)^2$.

$$\begin{aligned}(3a - 7b - c)^2 &= [3a + (-7b) + (-c)]^2 \\ &= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times 3a \\ &= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac\end{aligned}$$

(v) $(-2x + 5y - 3z)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

We need to apply the above identity to expand the expression $(-2x + 5y - 3z)^2$.

$$\begin{aligned}(-2x + 5y - 3z)^2 &= [(-2x) + 5y + (-3z)]^2 \\ &= (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \times (-2x) \times 5y + 2 \times 5y \times (-3z) + 2 \times (-3z) \times (-2x) \\ &= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx.\end{aligned}$$

(vi) $\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2$

We know that $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.

$$\begin{aligned}\left(\frac{1}{4}a - \frac{1}{2}b + 1\right)^2 &= \left[\frac{a}{4} + \left(-\frac{b}{2}\right) + 1\right]^2 \\ &= \left(\frac{a}{4}\right)^2 + \left(-\frac{b}{2}\right)^2 + (1)^2 + 2 \times \frac{a}{4} \times \left(-\frac{b}{2}\right) + 2 \times \left(-\frac{b}{2}\right) \times 1 + 2 \times 1 \times \frac{a}{4} \\ &= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}.\end{aligned}$$

Ex 2.4 Question 5.

Factorize:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Answer.

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

The expression $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ can also be written as

$$(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x$$

We can observe that, we can apply the identity $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with respect to the expression $(2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x$, to get $(2x + 3y - 4z)^2$

Therefore, we conclude that after factorizing the expression $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$, we get $(2x + 3y - 4z)^2$.

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

The expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$ can also be written as

$$(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x).$$

We can observe that, we can apply the identity $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with respect to the expression $(-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) + 2 \times (2\sqrt{2}z) \times (-\sqrt{2}x)$, to get $(-\sqrt{2}x + y + 2\sqrt{2}z)^2$

Therefore, we conclude that after factorizing the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$, we get $(-\sqrt{2}x + y + 2\sqrt{2}z)^2$.

Ex 2.4 Question 6.

Write the following cubes in expanded form:

(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $\left(\frac{3}{2}x + 1\right)^3$

(iv) $\left(x - \frac{2}{3}y\right)^3$

Answer.

(i) $(2x + 1)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$.

$$\begin{aligned} \therefore (2x + 1)^3 &= (2x)^3 + (1)^3 + 3 \times 2x \times 1(2x + 1) \\ &= 8x^3 + 1 + 6x(2x + 1) \\ &= 8x^3 + 12x^2 + 6x + 1. \end{aligned}$$

Therefore, the expansion of the expression $(2x + 1)^3$ is $8x^3 + 12x^2 + 6x + 1$.

(ii) $(2a - 3b)^3$

We know that $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$.

$$\begin{aligned} \therefore (2a - 3b)^3 &= (2a)^3 - (3b)^3 - 3 \times 2a \times 3b(2a - 3b) \\ &= 8a^3 - 27b^3 - 18ab(2a - 3b) \\ &= 8a^3 - 36a^2b + 54ab^2 - 27b^3. \end{aligned}$$

Therefore, the expansion of the expression $(2a - 3b)^3$ is $8a^3 - 36a^2b + 54ab^2 - 27b^3$.

(iii) $\left(\frac{3}{2}x + 1\right)^3$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$.

$$\begin{aligned} \left(\frac{3}{2}x + 1\right)^3 &= \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \frac{3}{2}x \times 1 \left(\frac{3}{2}x + 1\right) \therefore \\ &= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left(\frac{3}{2}x + 1\right) \\ &= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1 \end{aligned}$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x + 1\right)^3$ is $\frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$.

(iv) $\left(x - \frac{2}{3}y\right)^3$

We know that

$$\begin{aligned} \therefore \left(x - \frac{2}{3}y\right)^3 &= (x)^3 - \left(\frac{2}{3}y\right)^3 - 3 \times x \times \frac{2}{3}y \left(x - \frac{2}{3}y\right) \\ &= x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y\right) \\ &= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3. \end{aligned}$$

Therefore, the expansion of the expression $\left(x - \frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$.

$$x-y)^3=x^3-y^3-3 \times y(x-y).$$

Ex 2.4 Question 7.

Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Answer.

(i) $(99)^3$

$(99)^3$ can also be written as $(100 - 1)^3$.

Using identity, $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$(100 - 1)^3 = (100)^3 - (1)^3 - 3 \times 100 \times 1(100 - 1)$$

$$= 1000000 - 1 - 300(99)$$

$$= 999999 - 29700$$

$$= 970299$$

(ii) $(102)^3$

$(102)^3$ can also be written as $(100 + 2)^3$.

Using identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$$(100 + 2)^3 = (100)^3 + (2)^3 + 3 \times 100 \times 2(100 + 2)$$

$$= 1000000 + 8 + 600(102)$$

$$= 1000008 + 61200$$

$$= 1061208$$

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000 - 2)^3$.

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$$(1000 - 2)^3 = (1000)^3 - (2)^3 - 3 \times 1000 \times 2(1000 - 2)$$

$$= 1000000000 - 8 - 6000(998)$$

$$= 999999992 - 5988000$$

$$= 994011992$$

Ex 2.4 Question 8.

Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer.

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 + (b)^3 + 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b).$$

Using identity $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ with respect to the expression $(2a)^3 + (b)^3 + 3 \times 2a \times b(2a + b)$, we get $(2a + b)^3$.

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a + b)^3$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 - (b)^3 - 3 \times 2a \times 2a \times b + 3 \times 2a \times b \times b$$

$$= (2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression $(2a)^3 - (b)^3 - 3 \times 2a \times b(2a - b)$ we get $(2a - b)^3$.

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2$, we get $(2a - b)^3$.

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$$= (3)^3 - (5a)^3 - 3 \times 3 \times 3 \times 5a + 3 \times 3 \times 5a \times 5a$$

$$= (3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression $(3)^3 - (5a)^3 + 3 \times 3 \times 5a(3 - 5a)$, we get $(3 - 5a)^3$.

Therefore, after factorizing the expression $27 - 125a^3 - 135a + 225a^2$, we get $(3 - 5a)^3$.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 4a \times 3b + 3 \times 4a \times 3b \times 3b$$

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b).$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression $(4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$, we get $(4a - 3b)^3$.

Therefore, after factorizing the expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$, we get $(4a - 3b)^3$.

(v)

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$



The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$$\begin{aligned} &= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times 3p \times \frac{1}{6} + 3 \times 3p \times \frac{1}{6} \times \frac{1}{6} \\ &= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right) \end{aligned}$$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$ with respect to the expression

$$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6} \left(3p - \frac{1}{6}\right)$$

to get $\left(3p - \frac{1}{6}\right)^3$.

Therefore, after factorizing the expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$, we get $\left(3p - \frac{1}{6}\right)^3$.

Ex 2.4 Question 9.

Verify:

$$(i) \ x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$(ii) \ x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Answer.

$$(i) \ x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

We know that $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$.

$$\Rightarrow x^3 + y^3 = (x + y)^3 - 3xy(x + y)$$

$$= (x + y)[(x + y)^2 - 3xy]$$

$\therefore$ We know that $(x + y)^2 = x^2 + 2xy + y^2$

$$\therefore x^3 + y^3 = (x + y)(x^2 + 2xy + y^2 - 3xy)$$

$$= (x + y)(x^2 - xy + y^2)$$

Therefore, the desired result has been verified.

$$(ii) \ x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

We know that $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$.

$$\Rightarrow x^3 - y^3 = (x - y)^3 + 3xy(x - y)$$

$$= (x - y)[(x - y)^2 + 3xy]$$

$\therefore$ We know that $(x - y)^2 = x^2 - 2xy + y^2$

$$\therefore x^3 - y^3 = (x - y)(x^2 - 2xy + y^2 + 3xy)$$

$$= (x - y)(x^2 + xy + y^2)$$

Therefore, the desired result has been verified.

Ex 2.4 Question 10.

Factorize:

$$(i) \ 27y^3 + 125z^3$$

$$(ii) \ 64m^3 - 343n^3$$

Answer.

$$(i) \ 27y^3 + 125z^3$$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$.

$$(3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - 3y \times 5z + (5z)^2]$$

$$= (3y + 5z)(9y^2 - 15yz + 25z^2)$$

$$(ii) \ 64m^3 - 343n^3$$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

$$(4m)^3 - (7n)^3 = (4m - 7n)[(4m)^2 + 4m \times 7n + (7n)^2]$$

$$= (4m - 7n)(16m^2 + 28mn + 49n^2)$$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m - 7n)(16m^2 + 28mn + 49n^2)$.

Ex 2.4 Question 11.

Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Answer.

The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z.$$

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

$$\begin{aligned}\therefore (3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z &= (3x + y + z) [(3x)^2 + (y)^2 + (z)^2 - 3x \times y - y \times z - z \times 3x] \\ &= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz).\end{aligned}$$

Therefore, we conclude that after factorizing the expression $27x^3 + y^3 + z^3 - 9xyz$, we get $(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$.

Ex 2.4 Question 12.

Verify that

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$$

Answer.

LHS is $x^3 + y^3 + z^3 - 3xyz$ and RHS is $\frac{1}{2}(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

And also, we know that $(x - y)^2 = x^2 - 2xy + y^2$.

$$\begin{aligned}&\frac{1}{2}(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2] \\ &\frac{1}{2}(x + y + z) [(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2zx + x^2)] \\ &\frac{1}{2}(x + y + z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx) \\ &(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx).\end{aligned}$$

Therefore, we can conclude that the desired result is verified.

Ex 2.4 Question 13.

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 0$.

Answer.

We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

We need to substitute $x^3 + y^3 + z^3 = 0$ in $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$, to get

$$x^3 + y^3 + z^3 - 3xyz = (0) = (x^2 + y^2 + z^2 - xy - yz - zx),$$

$$x^3 + y^3 + z^3 - 3xyz = 0$$

$$\Rightarrow x^3 + y^3 + z^3 = 3xyz.$$

Therefore, the desired result is verified.

Ex 2.4 Question 14.

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3 + (7)^3 + (5)^3$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer.

(i) $(-12)^3 + (7)^3 + (5)^3$

Let $a = -12$, $b = 7$ and $c = 5$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = -12 + 7 + 5 = 0$

$$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$$

$$= -1260$$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$, $b = -15$ and $c = -13$

We know that, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

Here, $a + b + c = 28 - 15 - 13 = 0$

$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$

$$= 16380$$

Ex 2.4 Question 15.

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area: $25a^2 - 35a + 12$

(ii) Area: $35y^2 + 13y - 12$

Answer.

(i) Area : $25a^2 - 35a + 12$



The expression $25a^2 - 35a + 12$ can also be written as $25a^2 - 15a - 20a + 12$.

$$\begin{aligned} 25a^2 - 15a - 20a + 12 &= 5a(5a - 3) - 4(5a - 3) \\ &= (5a - 4)(5a - 3). \end{aligned}$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2 - 35a + 12$ is Length $= (5a - 4)$ and Breadth $= (5a - 3)$.

(ii) Area : $35y^2 + 13y - 12$

The expression $35y^2 + 13y - 12$ can also be written as $35y^2 + 28y - 15y - 12$.

$$\begin{aligned} 35y^2 + 28y - 15y - 12 &= 7y(5y + 4) - 3(5y + 4) \\ &= (7y - 3)(5y + 4). \end{aligned}$$

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13y - 12$ is Length $= (7y - 3)$ and Breadth $= (5y + 4)$.

Ex 2.4 Question 16.

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2 - 12x$

(ii) Volume: $12ky^2 + 8ky - 20k$

Answer.

(i) Volume : $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x - 4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is $3, x$ and $(x - 4)$.

(ii) Volume : $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$$\begin{aligned} k(12y^2 + 8y - 20) &= k(12y^2 - 12y + 20y - 20) \\ &= k[12y(y - 1) + 20(y - 1)] \\ &= k(12y + 20)(y - 1) \\ &= 4k \times (3y + 5) \times (y - 1). \end{aligned}$$

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k, (3y + 5)$ and $(y - 1)$.

